

U-duality (sub-)groups and their topology

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Abstract. We discuss some consequences of the fact that symmetry groups appearing in compactified (super-)gravity may be non-simply connected. The possibility to add fermions to a theory results in a simple criterion to decide whether a 3-dimensional coset sigma model can be interpreted as a dimensional reduction of a higher dimensional theory. Similar criteria exist for higher dimensional sigma models, though less decisive. Careful examination of the topology of symmetry groups rules out certain proposals for M-theory symmetries, which are not ruled out at the level of the algebra's. We conclude with an observation on the relation between the “generalized holonomy” proposal, and the actual symmetry groups resulting from E_{10} and E_{11} conjectures.

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1. Introduction

Since the construction of supergravities, and the discovery of the Cremmer-Julia groups of compactified 11 dimensional supergravities [1, 2] it has been clear that Lie groups and algebra's play an important role in this field.

In most treatments however, the attention is confined to Lie algebra's, and the global properties of the groups they generate are neglected. However, a study of these global properties may lead to useful information on the theory. In [3] some tools for the study of the topology of subgroups were given. Also two applications were discussed: A criterion which contains information on whether a theory is a dimensional reduction of a higher dimensional one; and, a critical examination (and unfortunately, falsification) of some proposals for symmetry groups of the yet elusive M-theory.

2. Topology of groups

We will not give a full discussion on the topology of groups here (the reader is referred to [3] and numerous textbooks on Lie groups), but only make a few remarks.

Every simple, compact Lie group G has a simply connected cover \tilde{G} . A fundamental theorem in Lie group theory states that the group G is isomorphic to \tilde{G}/Z , where Z is a subgroup of the center of \tilde{G} . The groups G and \tilde{G} have isomorphic Lie algebra's. Nevertheless, the effect of the center of \tilde{G} can be seen in representation theory.

An example that is well-known to the physicist is $SU(2)$. This group has a \mathbb{Z}_2 center, hence there are, up to isomorphism 2 different groups with Lie algebra $su(2)$, namely $SU(2)$ and $SO(3) \cong SU(2)/\mathbb{Z}_2$. There is exactly one irreducible representation (irrep) of $SU(2)$ of dimension n . If n is even, then this is an irrep of $SU(2)$, but not of $SO(3)$; the mapping of $SO(3)$ to an even dimensional irrep of $SU(2)$ is one-to-two and therefore not a homomorphism.

A similar relation is true for other compact Lie-groups: an irrep of \tilde{G} may not give an irrep of G . Unlike elsewhere in the physics literature, we will be precise in this paper; when we mention a group G , it is implied that all irreps that are irreps of \tilde{G} but not of G are absent. As an example, when we say that a symmetry group is $SO(3)$, it means that only odd-dimensional irreps are present.

An important fact is that, even if a group is simply connected, it may nevertheless have non-simply connected subgroups (the $SO(3)$ subgroup of $SU(3)$, obtained by restricting to *real* $SU(3)$ matrices is a simple example). The existence of such subgroups can lead to interesting physical effects [4], and is actually the crucial ingredient in our discussion below.

3. Fermions and oxidation

Consider a 3 dimensional sigma model on a coset G/H , coupled to gravity. An interesting question is whether this can be interpreted as the effective theory of the toroidal compactification of a higher dimensional theory. The answer to this question depends on the coset G/H , but is often affirmative [5, 6, 7, 8, 9]. In [9] we showed that the possibility for oxidation (reconstruction of the higher dimensional theory) can be deduced from properties of G , and that all possible higher dimensional theories are encoded in the geometry of the root lattice of G . Here instead, we will demonstrate that also H gives an immediate criterion about the possibility of oxidation.

Consider the possibility of adding fermions to the theory. The reduction of General Relativity from d to 3 dimensions gives rise to a 3-dimensional sigma model on $SL(d-2, \mathbb{R})/SO(d-2)$ [8]. The $SO(d-2)$ group appearing here can be thought of as the remnant of the helicity group in d dimensions [5]. We stress that the subgroup of $SL(d-2, \mathbb{R})$ is indeed $SO(d-2)$. Now $\pi_1(SO(d-2)) = \mathbb{Z}_2$ (for $d > 4$), a well known fact which is of course crucially related to the existence of fermions. Massless fermions in the higher dimensional theory transform in representations of $Spin(d-2)$ that are not representations of $SO(d-2)$. In the special case $d = 4$ we are dealing with $SO(2)$, and $\pi_1(SO(2)) = \mathbb{Z}$. Representations of $SO(2)$ are labelled by a number (spin), and it is customary to normalize this charge such that the bosons have integer spins. Then the fermions turn out to have half-integer spins, and again we are dealing with a double cover of the group relevant to the bosons.

The fact that the fermions transform in a double cover of the group remains true after dimensional reduction. But then it is crucial that the group H , appearing in the coset G/H must have a topology that is compatible with that of $SO(d-2)$. That is,

Table 1. Coset symmetries G/H for 3-d supergravity theories with N supersymmetries. For $N < 9$ there is extra freedom parametrized by an integer k . The maximal oxidation dimension is denoted by d .

N	d	G	H
16	11	$E_{8(8)}$	$Spin(16)/\mathbb{Z}_2$
12	6	$E_{7(-5)}$	$(Spin(12) \times SU(2))/\mathbb{Z}_2$
10	4	$E_{6(-14)}$	$Spin(10) \times U(1)$
9	3	$F_{4(-20)}$	$Spin(9)$
8, $k > 2$	$\min(10, k+2)$	$Spin(8, k)$	$(Spin(8) \times Spin(k))/\mathbb{Z}_2$
8, $k = 2$	4	$Spin(8, 2)$	$Spin(8) \times U(1)$
8, $k = 1$	3	$Spin(8, 1)$	$Spin(8)$
6, k	4	$SU(4, k)$	$S(U(4) \times U(k))$
5, k	3	$Sp(2, k)$	$Sp(2) \times Sp(k)$

the 2π rotation that leaves bosons invariant, but multiplies fermions with a sign, must be represented on H . In mathematical language

$$\pi_1(H) \supset \pi_1(SO(d-2)), \quad (1)$$

or, more precisely,

$$\pi_1(H) \supset \mathbb{Z} \text{ (for } d=4), \quad \pi_1(H) \supset \mathbb{Z}_2 \text{ (for } d>4). \quad (2)$$

This gives a *necessary* criterion for the possibility to oxidize. An analysis of the possibilities for oxidation from coset theories on G/H , with *simple* G , indicates that it is also a *sufficient* criterion [3]! Hence we have a

Theorem: Consider a sigma model in 3 dimensions on a symmetric space G/H , with G a simple non-compact group and H its maximal compact subgroup, coupled to gravity. This sigma model can be oxidized to a higher dimensional model if and only if the group H , as embedded in G , is not simply connected. Moreover, the maximal oxidation dimension d is given by:

$$\begin{aligned} d &= 3 \text{ if } \pi_1(H) = 0; \\ d &= 4 \text{ if } \pi_1(H) = \mathbb{Z}; \\ d &> 4 \text{ if } \pi_1(H) = \mathbb{Z}_2. \end{aligned} \quad (3)$$

A full list of cosets can be found in [3]. Here we restrict to examples that are related to cosets of 3 dimensional *supergravity* theories. For sufficiently many supersymmetries, the target space geometry of the sigma model must be a symmetric space [10]. A priori, the constraints of 3-dimensional supergravity are not related to spin (which does not exist in 3 dimensions). The theory of oxidation [9] and the considerations on fermions [3] provide the link between the analysis of [10], and the analysis in higher dimensional theories (which are restricted to have not more than one spin 2 excitation).

Table 1 was taken from [10] (but note a few corrections and the adaptation to our standards). Note: That our criterion confirms that theories with an odd number of 3-d supersymmetries cannot be oxidized (simply connected H); that theories which can be

oxidized to 4 dimensions have a single $u(1)$ factor in their H -algebra; and that for all theories that can be oxidized to higher dimensions (for which N is a multiple of 4, and which may require suitable matter content), the fundamental group of H is \mathbb{Z}_2 .

The reader may have noticed that a similar reasoning can be set up for theories in higher dimensions. Consider a d dimensional theory, with a sigma models on G/H . If the theory can be derived as a dimensional reduction of a yet higher-, $d+D$ dimensional theory, then the group H has to contain the group $SO(D)$, and moreover

$$\pi_1(H) \supset \pi_1(SO(D)). \quad (4)$$

It should be emphasized that, in contrast to the 3-dimensional case, this is really not more than a (rather weak) necessary criterion, as counterexamples to sufficiency are numerous (e.g. IIB supergravity in 10 dimensions, with $SL(2, \mathbb{R})/SO(2)$).

4. Generalized holonomy and symmetries of maximal supergravities

In [11, 12] a “generalized holonomy” proposal was put forward. The reasoning behind this proposal is roughly as follows.

There exist formulations of dimensionally reduced maximal supergravity with local symmetry $Spin(1, d-1) \times \tilde{H}_d$. Here $Spin(1, d-1)$ is obviously the local Lorentz-group. The second factor represents the double cover of a maximal compact subgroup of a Cremmer-Julia group [2] (see table 2 for a list of these). The existence of these “hidden” symmetries prompts the question whether these are a consequence of compactification, or already present in some form in the higher dimensional theory. An answer to this question was given in [14], where formulations of *11 dimensional supergravity* with local $Spin(1, d-1) \times \tilde{H}_d$ invariance were constructed.

These symmetries are local, and presumably also symmetries of the proposed non-perturbative extension of 11-d supergravity, M-theory. Upon compactification, such symmetries are broken by boundary conditions; more accurately, there is non-trivial *holonomy* in the group $Spin(1, d-1) \times \tilde{H}_d$, such that it is no longer a manifest symmetry of the lower dimensional theory.

The groups $Spin(1, d-1) \times \tilde{H}_d$ refer to a specific factorization of the background geometry, into a d -dimensional part, containing the time-like direction, and an $(11-d)$ dimensional part. For a full description, one wants to know \tilde{H}_d for all values of d , specifically for $d = 0$. For $d \geq 3$, these groups are known from the Cremmer-Julia analysis [2], and [14]. For $d = 2, 1$ the groups $Spin(16) \times Spin(16)$ and $Spin(32)$ were proposed by [11], for $d = 0$ a proposal is $SL(32, \mathbb{R})$ [12]. We will here re-examine these proposals more carefully.

In table 2 we have collected the Cremmer-Julia groups, and their actual compact subgroups. Our table differs from many others in the literature because we have been careful to mention the compact groups H_d with their correct topologies. The reader will notice that for $d < 8$, all H_d are simple, and *two-fold connected*.

The two-fold connectedness is related, as before, to fermionic representations. The bosons in the supergravity theories transform in irreps of H_d . The fermions however,

Table 2. For $d \geq 3$: Cremmer-Julia groups G_d ; their compact subgroups H_d . For $d < 3$: Candidate “generalized holonomy”-groups in lower dimensions

d	G_d	H_d
11	$\{e\}$	$\{e\}$
10	$\mathbb{R}, SL(2, \mathbb{R})$	$\{e\}, SO(2)$
9	$SL(2, \mathbb{R}) \times \mathbb{R}$	$SO(2)$
8	$SL(3, \mathbb{R}) \times SL(2, \mathbb{R})$	$SO(3) \times SO(2)$
7	$SL(5, \mathbb{R})$	$SO(5)$
6	$Spin(5, 5)$	$(Sp(2) \times Sp(2))/\mathbb{Z}_2$
5	$E_{6(6)}$	$Sp(4)/\mathbb{Z}_2$
4	$E_{7(7)}$	$SU(8)/\mathbb{Z}_2$
3	$E_{8(8)}$	$Spin(16)/\mathbb{Z}_2$
2		$Spin(16) \times Spin(16)$
1		$Spin(32)$
0		$SL(32, \mathbb{R})$

transform in irreps of the double cover \tilde{H}_d that are not irreps of H_d . An important fact to keep in mind is that, since \tilde{H}_d is not a subgroup of G , G can represent at most symmetries from the bosonic sector of the theory. Not only do the fermions not transform in irreps of G , there does not even exist a G representation that has the fermionic irreps in its H_d decomposition.

Nevertheless, since there are fermions present in the theory, the full symmetry of the theory contains \tilde{H}_d . The example that will be important to us is 3-dimensional maximal supergravity [15]. The compact subgroup of $E_{8(8)}$ is $Spin(16)/\mathbb{Z}_2$. The scalars in the theory are in the **128_s**, which is an irrep of $Spin(16)/\mathbb{Z}_2$. The (non-dynamical) gravitini are in the **16**, whereas the remaining fermions are in the **128_c** (the *other* spin irrep of $Spin(16)$). Neither of the latter 2 irreps is an irrep of $Spin(16)/\mathbb{Z}_2$, and hence the full symmetry of the theory is $\tilde{H}_3 = Spin(16)$.

We now turn to the proposed generalized holonomy groups for $d < 3$. By restriction to a smaller number of “internal” dimensions, we expect

$$\tilde{H}_d \supset \tilde{H}_{d+1}. \quad (5)$$

Hence, we expect

$$SL(32, \mathbb{R}) \supset Spin(32) \supset Spin(16) \times Spin(16) \supset Spin(16). \quad (6)$$

The crucial point however is that equation 6 is *false!* The actual subgroup of $SL(32, \mathbb{R})$ with $so(32)$ algebra is $SO(32)$, and not $Spin(32)$: No spin irreps of $Spin(32)$ can appear in the decomposition from $SL(32, \mathbb{R})$ irreps. In turn, the subsequent subgroup of $SO(32)$ is $SO(16) \times SO(16)$, and this subgroup has only $SO(16)$ subgroups. It is impossible to obtain the irreps **128_s** and **128_c** of the scalars and fermions in 3-d supergravity, from any $SL(32, \mathbb{R})$ irrep.

Some more thought reveals that $SL(32, \mathbb{R})$ has no $Spin(16)$ subgroups whatsoever [3] also if we are willing to give up the chain in equation 6. The inevitable conclusion is

then that 3-d supergravity has symmetries not contained in $SL(32, \mathbb{R})$ (related to the center of $Spin(16)$) which can therefore not be a symmetry group in the sense proposed in [12].

A similar, but slightly more subtle reasoning applies to $Spin(32)$. The subgroup of $Spin(32)$ with $so(16) \oplus so(16)$ algebra is $(Spin(16) \times Spin(16))/\mathbb{Z}_2$. This group has various subgroups with $so(16)$ algebra. There are essentially two options, an embedding in one of the two factors, or the diagonal one. The proposal in [11] claims that the 2-d gravitini should be in the $(\mathbf{16}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{16})$. Then, to get the proper **16** for the 3-d gravitini, we should select the diagonal embedding. But the diagonal subgroup in $(Spin(16) \times Spin(16))/\mathbb{Z}_2$ is $SO(16)$, and we again find a contradiction. Alternatively, a non-diagonal embedding leads to 16 singlets that do not fit in the 3-d theory, that furthermore would transform in an (unobserved) extra $Spin(16)$ factor. As it seems impossible to make sense out of this, we discard $Spin(32)$ as candidate group for \tilde{H}_1 .

We cannot rule out $Spin(16) \times Spin(16)$ on the basis of these arguments, but it is clear that there is reason to distrust this group too. Indeed, a careful analysis of 2-d maximal supergravity, as performed in [18] does not indicate this symmetry.

Though the abstract “generalized holonomy” proposal is attractive, the precise symmetry groups proposed seem to be ruled out. It is not easy to find alternative, finite-dimensional candidates. Instead, conjectures on E_{10} and E_{11} symmetries in maximal supergravity seem to indicate that the local symmetry groups should be infinite dimensional. The next section demonstrates a link between these infinite dimensional groups, and the discarded “generalized holonomy groups”.

5. Generalized holonomy and infinite dimensional groups

In spite of the fact that the proposal that these groups are symmetry groups for M-theory turns out to be untenable, there is a very simple relation between the groups proposed, and the $E_{n(n)}$ groups of maximal supergravities, also for $n = 10, 11$ ($d = 1, 0$), where the symmetry groups have a conjectural status.

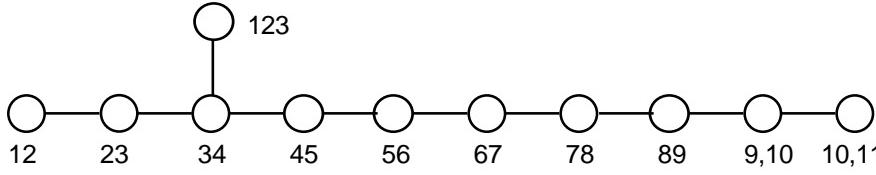


Figure 1. The Dynkin diagram of E_{11} .

In figure 1 we have depicted the Dynkin diagram of E_{11} , but the discussion below extends to any E_n group with $n < 11$ by suitably truncating the Dynkin diagram (it also extends to E_n with $n > 11$, but this is without obvious application to supergravity). We have labelled all nodes with a set of integers, the nodes along the horizontal line with a pair of integers, the branch with a triplet of integers.

The “generalized holonomy groups” from [11, 12] are obtained as follows. To obtain the relevant group for d dimensions, we omit all nodes that have numbers larger than d in their index set from the diagram. For each of the remaining nodes, we form the Clifford algebra element Γ^{S_i} , where S_i is the index set coming with node i . As usual Γ^{S_i} is a product of gamma matrices, fully anti-symmetrized in the indices. Next we form the algebra of consecutive commutators of the Γ^{S_i} . This algebra in turn generates a Lie group H_Γ , which is the “generalized holonomy group” mentioned in [11, 12]. The groups for time-like, space-like and null reduction follow from including in the set of generating gamma matrices an element that squares to $-1, 1, 0$, respectively.

The reason for presenting the “generalized holonomy groups” like this, is that the above construction has clear parallels with the abstract construction of Lie algebra’s [16]. Also there the Lie algebra is defined by forming consecutive commutators of ladder operators e_i . Another set of generators consists of consecutive commutators of the conjugate ladder operators f_i . Together with the Cartan generators h_i , the e_i and f_i generate the full algebra.

The factor group H appearing in $E_{n(n)}/H$ is generated by elements of the form $e_i - \epsilon_i f_i$, and their commutators. The $\epsilon_i = 1, -1, 0$ is included to allow for other than spacelike reductions (if a null or timelike direction is present we choose this to be the d -direction; $\epsilon_i = 0(-1)$ for a node i if the index set associated to it contains a null (timelike) direction; otherwise $\epsilon_i = 1$.).

For *finite-dimensional* $E_{n(n)}$ we have *exactly*

$$H \cong H_\Gamma. \quad (7)$$

There is a one-to-one relationship between $e_i - \epsilon_i f_i$ and Γ^{S_i} . Furthermore $\epsilon_i = -(\Gamma^{S_i})^2$ (where the left hand side includes the identity on the spinor algebra).

This is however not so for $n > 8$. The group H_Γ is always a *finite-dimensional* group, while for $n > 8$ the group H is clearly infinite. Nevertheless, there is still the relation between the generating elements of H , and the Γ^{S_i} . If we, as in [11], denote symbolically by $\Gamma^{(n)}$ the Clifford algebra elements obtained by multiplying n gamma-matrices and anti-symmetrization in the indices, then we have:

$$\begin{aligned} [\Gamma^{(3)}, \Gamma^{(3)}] &= \Gamma^{(2)} + \Gamma^{(6)}; & [\Gamma^{(3)}, \Gamma^{(6)}] &= \Gamma^{(3)} + \Gamma^{(7)}; \\ [\Gamma^{(3)}, \Gamma^{(7)}] &= \Gamma^{(6)} + \Gamma^{(10)}; & [\Gamma^{(3)}, \Gamma^{(10)}] &= \Gamma^{(7)} + \Gamma^{(11)}; \dots \end{aligned} \quad (8)$$

In terms of the level expansion by the “exceptional root” (as proposed in [17]), Γ^{123} corresponds to a linear combination of a ladder generator e_k at level 1 and one f_k at level -1 . Similarly, all $\Gamma^{(3)}$ correspond to level ± 1 generators, $\Gamma^{(6)}$ to level ± 2 generators, $\Gamma^{(7)}$ to level ± 3 generators, and so on. It is now easy to see that the algebra generated by the Γ^{S_i} corresponds to the algebra H truncated beyond level $\pm(2k-1)$ for $n < (4k+2)$, and beyond level $\pm 2k$ for $n < (4k+3)$. Furthermore, $SO(n)$ -representations other than completely antisymmetric tensors are *excluded* (because the Clifford property of gamma-matrices contracts all symmetrized indices).

For $n < 9$ the level truncation imposes no restriction and all irreps are antisymmetric tensors (see [9]). For $n = 11$, one precisely finds the generators

mentioned in [13] (with additional ones that are actually needed to complete the group to $SL(32, \mathbb{R})$). It should be stressed however, that $sl(32, \mathbb{R})$ is *not* a sub-algebra of e_{11} (nor is $so(32)$ a subalgebra of e_{10} , or $so(16) \oplus so(16)$ a sub-algebra of e_9): The truncation implied by the Γ^{S_i} is an *illegal* procedure when selecting sub-algebra's!

Whether the relation exhibited here has any other profound consequences, we leave for future research and speculation.

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